B-Modes from Scalar-Induced Gravitational Waves

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Mode mixing effects

- Start with scalar and tensor modes only (comment: these are the only ones that can be generated from vacuum fluctuations in GR).
- At second order (and beyond):
- Second-order scalars induce:
  - Small-scale Newtonian (and beyond) and large-scale GR non-linearities
  - Scalar-induced vector modes → frame-dragging → vorticity in multi-fluids → magnetic fields / B-mode polarization
  - Scalar-induced GW / B-mode polarization
- Second-order tensors induce:
  - Tensor-induced scalar modes → scalar perts. during inflation / DM density perts.
  - Tensor-induced vector modes
  - Tensor-induced tensor modes

New: Account for quantum particle production due to scalar (and tensor) inhomogeneities (see Hu & Verdaguer 2020 for a review) also in the case of <u>Weyl invariant fields</u>: Mierna, Perna, Bartolo, Matarrese & Ricciardone, *to appear soon.*

Intrinsic B-modes

- Scalar-induced vectors and tensors give rise to an ubiquitous B-mode polarization contribution Mollerach, Harari & Matarrese (2004) computed the contribution to the polarization of the CMB induced by vector and tensor modes generated by the nonlinear evolution of primordial scalar perturbations. Our calculation is based on relativistic second-order perturbation theory and allows to estimate the effects of these secondary modes on the polarization angular power spectra. A non-vanishing B-mode polarization unavoidably arises from pure scalar initial perturbations. This secondary effect dominates over that of primordial tensors for an inflationary tensor-to-scalar ratio $r < 10^{-6}$. The magnitude of the effect is smaller than the contamination produced by the conversion of polarization of type E into type B, by weak gravitational lensing. Cleaning?

Scalar-induced vector & tensor modes (Matarrese et al. 1997)

$$ds^2=a^2(\eta)\{-(1+2\Psi)d\eta^2-2V_i d\eta dx^i$$
$$+[(1-2\Phi)\delta_{ij}+2H_{ij}]dx^i dx^j\},$$

Scalar-induced vector modes

$$\nabla^2\nabla^2 V_i=16\pi G a^2\bar{\rho}\partial^j(v_{Pj}\partial_i\delta_P-v_{Pi}\partial_j\delta_P).$$

$$\delta_P=\frac{1}{4\pi G a^2\bar{\rho}}[\nabla^2\varphi-3\mathcal{H}(\dot{\varphi}+\mathcal{H}\varphi)],$$

$$v_{Pi}=-\frac{1}{4\pi G a^2\bar{\rho}}\partial_i(\dot{\varphi}+\mathcal{H}\varphi).$$

$$\ddot{\varphi}+3\mathcal{H}\dot{\varphi}+a^2\Lambda\varphi=0.$$

Scalar-induced tensor modes

$$H_{Sij}(\mathbf{x},\eta)=\frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot\mathbf{x}}\frac{40}{k^4}\mathcal{S}_{ij}(\mathbf{k})\left(\frac{1}{3}-\frac{j_1(k\eta)}{k\eta}\right),$$

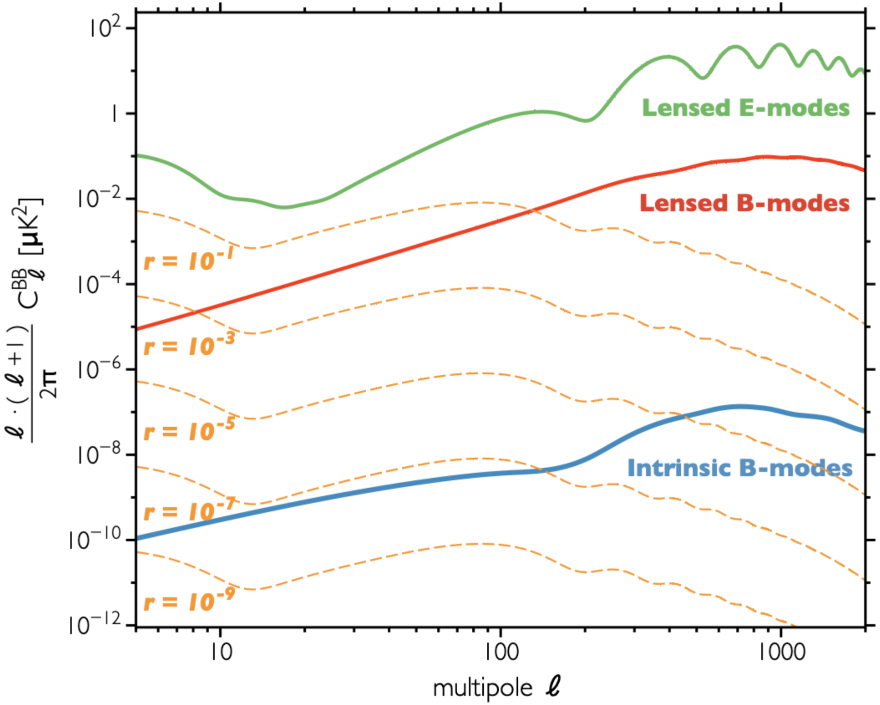
$$\mathcal{S}_{ij}=\nabla^2\Theta_0\delta_{ij}+\partial_i\partial_j\Theta_0+2(\partial_i\partial_j\varphi_0\nabla^2\varphi_0-\partial_i\partial_k\varphi_0\partial^k\partial_j\varphi_0).$$

$$\nabla^2\Theta_0=-\frac{1}{2}[(\nabla^2\varphi_0)^2-\partial_i\partial_k\varphi_0\partial^i\partial^k\varphi_0].$$

Selecting only the growing-mode solution, we can write $\varphi(\mathbf{x},\eta)=\varphi_0(\mathbf{x})g(\eta)$, where φ_0 is the peculiar gravitational potential linearly extrapolated to the present time (η_0) and $g=D_+/a$ is the so-called growth-suppression factor, where $D_+(\eta)$ is the linear growing-mode of density fluctuations in the Newtonian limit and a the scale-factor.

Intrinsic B-mode contribution

In a later calculation based on the second-order Boltzmann code SONG, Fidler et al. (2014) find this contamination to be comparable to a primordial tensor-to-scalar ratio of $r \simeq 10^{-7}$ at the angular scale $\ell \simeq 100$, where the primordial signal peaks, and $r \simeq 5 \times 10^{-5}$ at $\ell \simeq 700$, where the intrinsic signal peaks.



Can this effect be more significant?

A viable possibility could be that of increasing the power of scalar modes on small scales (as in all current PBH models), so as to increase the overall weight of the internal loop (Perna et al. in progress).

- Indeed, for many years, based on extrapolation of CMB and galaxy scale observational data we assumed limited power both for scalar and tensor modes.

- In the last ~ 10 years our attitude changed, both for small-scale scalar modes (e.g. motivated by PBH formation) and large-scale tensor modes (e.g. motivated by axion-inflation models).

Can this mechanism work? Any alternative enhancement mechanisms acting on CMB scales?

Effects of delayed/rapid reheating

- It has recently pointed out that non-instantaneous reheating can induce a resonant-like enhancement of the SIGW signal on large and intermediate frequency (e.g. Inomata et al. 2019; Pierce et al. 2023; …). An epoch of matter domination in the early universe can enhance SIGW, potentially making it detectable to upcoming gravitational wave experiments. However, the resulting gravitational wave signal is quite sensitive to the end of the early matter era. If matter domination ends gradually, a cancellation results in an extremely suppressed signal, while in the limit of an instantaneous transition, there is a resonant-like enhancement.

- Enhancement on small/intermediate scales, but not on large scales.

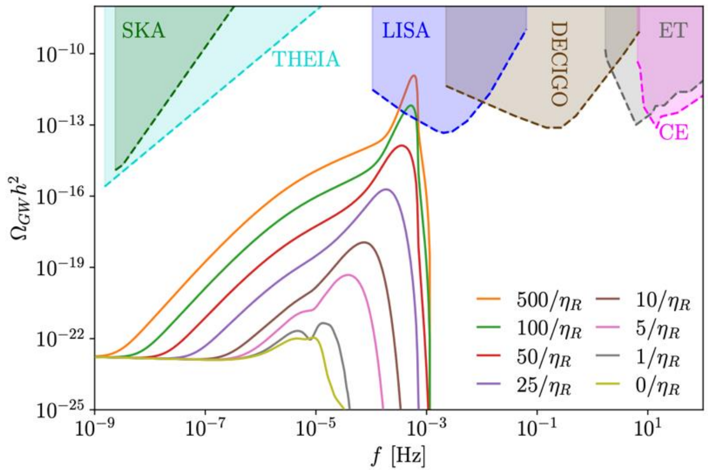


Figure 7. The GW signal induced from a transition at $T_R = 100$ GeV with a realistic curvature power spectrum as in eq. (2.3). The signals are labelled by the value of β. We see only the $\beta = 500/\eta_R$ and $\beta = 100/\eta_R$ spectra would be detectable at LISA. We compare to sensitivity curves of future GW experiments for LISA observing for four years [6], the Einstein Telescope observing for one year [49, 53, 54], the Cosmic Explorer [55], DECIGO with three units observing for one year [53, 56], THEIA observing for twenty years [57, 58] and SKA [59].

Decay rate parametrization: $\Gamma(\eta) = \Gamma_{max}(\tanh(\beta(\eta - \eta_\Gamma)) + 1)/2.$

Pierce et al. 2023

Spatial Intermittency in the CGWB

- Mierna, Matarrese, Bartolo & Ricciardone 2024 showed that the large-scale SW effect for the Cosmological Gravitational-Wave background produced during inflation is lognormal distributed if teh GW power-spectrum is (nearly) scale-invariant (n_T=0) and the scalar perturbations are (nearly) Gaussian

Curvature perturbation (nearly Gaussian)

$$\delta_{\rm GW} = A\, e^{-\zeta} - 1$$

CGWB energy density fluctuation (lognormal).

- Lognormal distributions are characterized by "intermittency", (e.g. Zel'dovich et al. 1987), which consists in the *emergence of isolated high-density spots surrounded by large underdense areas*. This makes the primordial cosmological signal distinguishable from the astrophysical one.

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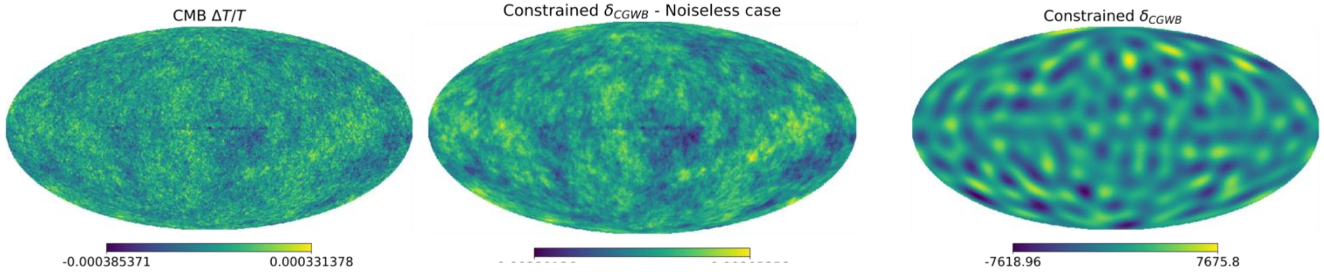
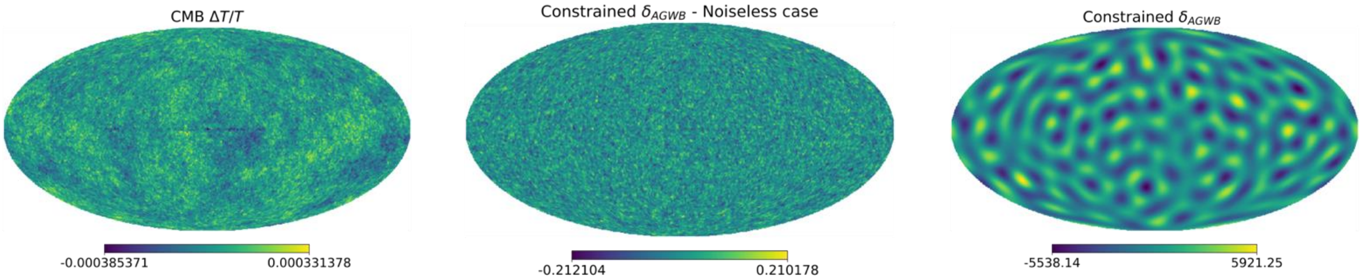
Constrained realizations

CGWB x CMB Planck SMICA map



$$h''_\lambda(\tau,\mathbf{k}) + 2\mathcal{H}h'_\lambda(\tau,\mathbf{k}) + k^2 h_\lambda(\tau,\mathbf{k}) = 4\mathcal{S}_\lambda(\tau,\mathbf{k}),$$

AGWB x CMB Planck SMICA map



Ricciardone, Valbusa, Bartolo, Bertacca, Liguori, Matarrese 2021, PRL **127** 271301

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Conclusions

- Mode mixing of cosmological perturbations leads to several novel phenomena, like scalar-induced vector and tensor modes, tensor-induced scalar modes, etc. … (Tomita 1967; Matarrese et al. 1993, 1998; Ananda et al. 2007; Baumann et al. 2007; … Bari et al. 2023; Kugarajh et al. 2025; Abdelaziz et al. 2025).

- Although these effects were originally thought to be tiny, our attitude recently changed; e.g. Figueroa et al. (2024) showed that scalar-induced tensor modes are a viable candidate to source the PTA signal observed by NANOgrav etc. … and calculatios including delayed/rapid reheating lead to very promising predictions.

- In the extreme case (Bertacca, Jimenez, Matarrese & Ricciardone 2024) one can figure out a scenario where tensor metric fluctuations (GW) naturally arise from quantum vacuum oscillations, while scalar fluctuations are generated via second-order tensors only.